Department of Mathematics & Philosophy of Engineering

Faculty of Engineering Technology

The Open University of Sri Lanka

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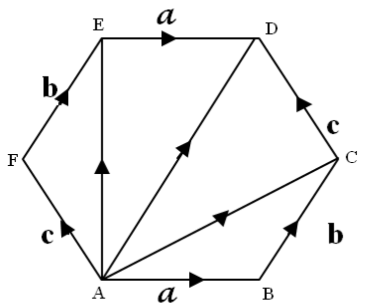
Academic Year:	2012/2013	Assignment No.	01
Course Code:	MPZ3231		

(1) (a)



(i) $\overrightarrow{AB} = \boldsymbol{a}$ $\quad = \boldsymbol{a}$

$\overrightarrow{AC} = \boldsymbol{a} + \boldsymbol{b}$ $\quad = \boldsymbol{a} + \boldsymbol{b}$

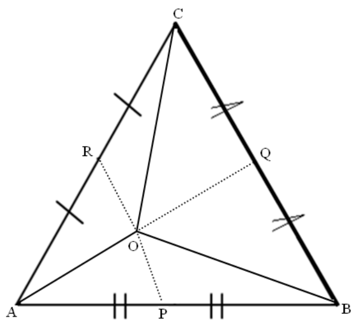
$\overrightarrow{AD} = \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = 2\boldsymbol{b}$

$\overrightarrow{AE} = \boldsymbol{b} + \boldsymbol{c}$ $\quad = 2\boldsymbol{b} - \boldsymbol{a}$

$\overrightarrow{AF} = \boldsymbol{c}$ $\quad = \boldsymbol{b} - \boldsymbol{a}$

(ii) $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 3[\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}] \quad = 3\overrightarrow{AD}$

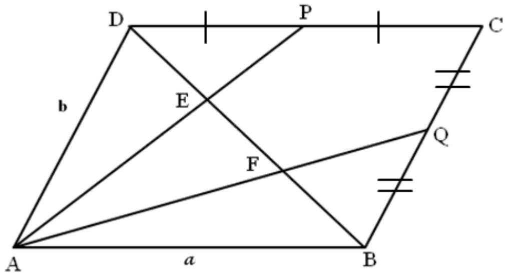
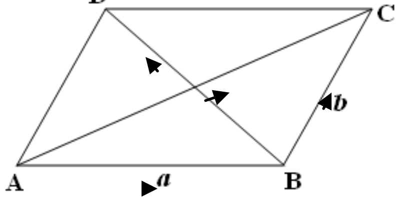
(b)



$\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$

$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OC}$

$\overrightarrow{OP} = \overrightarrow{OA} + {}^{1}/_{2}\,\overrightarrow{AB}$ $\overrightarrow{OQ} = \overrightarrow{OB} + {}^{1}/_{2}\,\overrightarrow{BC}$ $\overrightarrow{OR} = \overrightarrow{OA} + {}^{1}/_{2}\,\overrightarrow{AC}$ $\overrightarrow{OP} + \overrightarrow{OQ} + \overrightarrow{OR}$ $= \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OA} + {}^{1}/_{2}\,[\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{AC}]$ $= \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OA} + {}^{1}/_{2}\,[\overrightarrow{AC} + \overrightarrow{AC}]$ $= \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OA} + \overrightarrow{AC}$ $= \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$ (c)  it is clear that the line from A to the midpoint of CD is represented by the vector $\boldsymbol{a}/2 + \boldsymbol{b}$. A vector drawn from O to any point on this line can be written as $\mathbf{r}(\lambda) = \lambda\,(\boldsymbol{a}/2 + \boldsymbol{b})$. The diagonal BD is represented by the vector the vector $\mathbf{b} - \boldsymbol{a}$. A vector drawn from O to any point on this diagonal is $\mathbf{r}(\mu) = \boldsymbol{a} + \mu\,(\mathbf{b} - \boldsymbol{a})$, where the parameter μ adjusts the length of the diagonal. The two lines meet when $\lambda\,(\boldsymbol{a}/2 + \boldsymbol{b}) = \boldsymbol{a} + \mu\,(\mathbf{b} - \boldsymbol{a})$, which can be written as $(\lambda/2 - 1 + \mu)\,\boldsymbol{a} + (\lambda - \mu)\,\mathbf{b} = \mathbf{0}$ This gives $\mu = {}^{2}/_{3}$ and $\lambda = {}^{2}/_{3}$ so the length of BE is two-thirds of BD. Similarly, we can show the length FD is two-thirds of BD. (d) (i)  $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ $\qquad \overrightarrow{AC} = \boldsymbol{a} + \boldsymbol{b}$ $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$ $\qquad \overrightarrow{BD} = \boldsymbol{b} - \boldsymbol{a}$	

$\overrightarrow{AC} + \overrightarrow{BD} = \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{b} - \boldsymbol{a}$

$\overrightarrow{AC} + \overrightarrow{BD} = 2\boldsymbol{b}$

$\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{BC}$

(ii) $\overrightarrow{AC} - \overrightarrow{BD} = \boldsymbol{a} + \boldsymbol{b} - (\boldsymbol{b} - \boldsymbol{a})$

$\overrightarrow{AC} - \overrightarrow{BD} = 2\boldsymbol{a}$

$\overrightarrow{AC} - \overrightarrow{BD} = 2\overrightarrow{AB}$

(2) (a) Let $\boldsymbol{a}$, $\boldsymbol{b}$ two vectors

Scalar product

If $\boldsymbol{a} \neq 0$ and $\boldsymbol{b} \neq 0$ then $\boldsymbol{a} \,.\, \boldsymbol{b} = |\boldsymbol{a}|\ |\boldsymbol{b}|\ \mathbf{cos}\,\theta$ **,** where θ is the angle between $\boldsymbol{a}$, $\boldsymbol{b}$

If $\boldsymbol{a} = 0$ or $\boldsymbol{b} = 0$ then $\boldsymbol{a} \,.\, \boldsymbol{b} = 0$

(b) $(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})^2 = 0$

$0 = \boldsymbol{a}\,.\,\boldsymbol{a} + \boldsymbol{a}\,.\,\boldsymbol{b} + \boldsymbol{a}\,.\,\boldsymbol{c} + \boldsymbol{b}\,.\,\boldsymbol{a} + \boldsymbol{b}\,.\,\boldsymbol{b} + \boldsymbol{b}\,.\,\boldsymbol{c} + \boldsymbol{c}\,.\,\boldsymbol{a} + \boldsymbol{c}\,.\,\boldsymbol{b} + \boldsymbol{c}\,.\,\boldsymbol{c}$

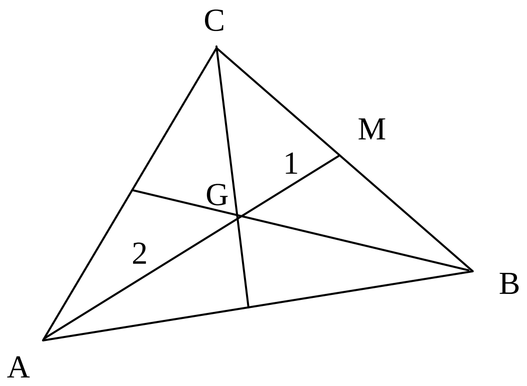
$0 = 3^2 + 4^2 + 5^2 + 2\boldsymbol{a}\,.\,\boldsymbol{b} + 2\boldsymbol{a}\,.\,\boldsymbol{c} + 2\boldsymbol{b}\,.\,\boldsymbol{c}$

$0 = 50 + 2(\boldsymbol{a}\,.\,\boldsymbol{b} + \boldsymbol{a}\,.\,\boldsymbol{c} + \boldsymbol{b}\,.\,\boldsymbol{c})$

$-50 = 2(\boldsymbol{a}\,.\,\boldsymbol{b} + \boldsymbol{a}\,.\,\boldsymbol{c} + \boldsymbol{b}\,.\,\boldsymbol{c})$

$(\boldsymbol{a}\,.\,\boldsymbol{b} + \boldsymbol{a}\,.\,\boldsymbol{c} + \boldsymbol{b}\,.\,\boldsymbol{c}) = -25$

(c)



Consider $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

$\left(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}\right) \cdot \left(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}\right) = 0$

$\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{CA}\right|^2 + \overrightarrow{AB} \cdot \overrightarrow{BC} + \overrightarrow{AB} \cdot \overrightarrow{CA} + \overrightarrow{BC} \cdot \overrightarrow{AB} + \overrightarrow{BC} \cdot \overrightarrow{CA} + \overrightarrow{CA} \cdot \overrightarrow{AB} + \overrightarrow{CA} \cdot \overrightarrow{BC} = 0$

But, $\overrightarrow{AB} \cdot \overrightarrow{BC} = \overrightarrow{BC} \cdot \overrightarrow{AB}$, $\overrightarrow{AB} \cdot \overrightarrow{CA} = \overrightarrow{CA} \cdot \overrightarrow{AB}$, $\overrightarrow{BC} \cdot \overrightarrow{CA} = \overrightarrow{CA} \cdot \overrightarrow{BC}$

$\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{CA}\right|^2 + 2(\overrightarrow{AB} \cdot \overrightarrow{BC}) + 2(\overrightarrow{AB} \cdot \overrightarrow{CA}) + 2(\overrightarrow{BC} \cdot \overrightarrow{CA}) = 0$ —— (1)

But, $\overrightarrow{BG}=\frac{2\overrightarrow{BM}+\overrightarrow{BA}}{3}=\frac{2\cdot\frac{1}{2}\overrightarrow{BC}+\overrightarrow{BA}}{3}$; $\overrightarrow{BM}=\frac{1}{2}\overrightarrow{BC}$

$$3\overrightarrow{BG}=\overrightarrow{BC}+\overrightarrow{BA}$$

similarly, $3\overrightarrow{AG}=\overrightarrow{AB}+\overrightarrow{AC}$,

$$3\overrightarrow{CG}=\overrightarrow{CB}+\overrightarrow{CA}$$

$$3\overrightarrow{BG}\cdot3\overrightarrow{BG}=\left(\overrightarrow{BC}+\overrightarrow{BA}\right)\cdot\left(\overrightarrow{BC}+\overrightarrow{BA}\right)$$

$$9BG^2=BC^2+BA^2+2\overrightarrow{BA}\cdot\overrightarrow{BC}=BC^2+BA^2-2\overrightarrow{AB}\cdot\overrightarrow{BC}$$

$$9AG^2=AB^2+AC^2+2\overrightarrow{AB}\cdot\overrightarrow{AC}=AB^2+AC^2-2\overrightarrow{AB}\cdot\overrightarrow{CA}$$

$$9CG^2=CB^2+CA^2+2\overrightarrow{CB}\cdot\overrightarrow{CA}=CB^2+CA^2-2\overrightarrow{BC}\cdot\overrightarrow{CA}$$

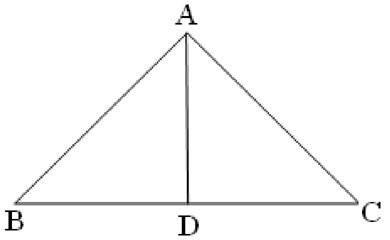
from (1), $AB^2+BC^2+CA^2=-2\left(\overrightarrow{AB}\cdot\overrightarrow{BC}\right)-2\left(\overrightarrow{AB}\cdot\overrightarrow{CA}\right)-2\left(\overrightarrow{BC}\cdot\overrightarrow{CA}\right)$

$$AB^2+BC^2+CA^2=9BG^2-BC^2-BA^2+9AG^2-AB^2-AC^2+9CG^2-CB^2-CA^2$$

$$3(AB^2+BC^2+CA^2)=9(BG^2+AG^2+CG^2)$$

$$(AB^2+BC^2+CA^2)=3(BG^2+AG^2+CG^2)$$

(d)



$$\overrightarrow{AB}=\overrightarrow{AD}+\overrightarrow{DB}$$

$$AB^2=(\overrightarrow{AD}+\overrightarrow{DB})^2$$

$$=AD^2+DB^2+2.\overrightarrow{AD}.\overrightarrow{DB}\ \text{----------}(1)$$

Also we have

$$\overrightarrow{AC}=\overrightarrow{AD}+\overrightarrow{DC}$$

$$AC^2=(\overrightarrow{AD}+\overrightarrow{DC})^2$$

$$=AD^2+DC^2+2.\overrightarrow{AD}.\overrightarrow{DC}\ \text{----------}(2)$$

(1)+(2)

$$AB^2+AC^2=2AD^2+2BD^2+2.\overrightarrow{AD}.(\overrightarrow{DB}+\overrightarrow{DC})$$

$=2(AD^2+BD^2)$, for $\overrightarrow{DB}+\overrightarrow{DC}=0$

(3). (a) Let $\boldsymbol{a}$, $\boldsymbol{b}$ two vectors

Vector product

If $\boldsymbol{a} \neq 0$ and $\boldsymbol{b} \neq 0$ then $\boldsymbol{a} \times \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\boldsymbol{sin\,\theta\, n}$ **,**

where θ is the angle between $\boldsymbol{a}$, $\boldsymbol{b}$

The direction is that of the unit vector $\boldsymbol{n}$ which is perpendicular to both $\boldsymbol{a}$, $\boldsymbol{b}$

such that $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{n}$ form a right handed system.

If $\boldsymbol{a} = \boldsymbol{0}$ or $\boldsymbol{b} = \boldsymbol{0}, \boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{0}$

(b) $(\boldsymbol{a} + \mu\boldsymbol{b}) \cdot (\boldsymbol{a} - \mu\boldsymbol{b}) = 0$

$(\boldsymbol{a} \cdot \boldsymbol{a}) - \boldsymbol{a} \cdot \mu\boldsymbol{b} + \mu\boldsymbol{b} \cdot \boldsymbol{a} + \mu\boldsymbol{b} \cdot (-\mu\boldsymbol{b}) = 0$

$|\boldsymbol{a}|^2 - \mu\boldsymbol{a} \cdot \boldsymbol{b} + \mu\boldsymbol{a} \cdot \boldsymbol{b} - \mu^{\boldsymbol{2}}|\boldsymbol{b}|^{\boldsymbol{2}} = 0$

$3^2 - \mu^2 4^2 = 0$

$\frac{3^2}{4^2} = \mu^2 \Rightarrow \mu = \pm\frac{3}{4}$

(c) If collinear then parallel, $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{0}$

Let $\boldsymbol{a} = -3\boldsymbol{i} + 4\boldsymbol{j} + \mu\boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{\lambda i} + 8\boldsymbol{j} + 6\boldsymbol{k}$

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -3 & 4 & \mu \\ \lambda & 8 & 6 \end{vmatrix} = \boldsymbol{i}(24 - 8\mu) - \boldsymbol{j}(-18 - \lambda\mu) + \boldsymbol{k}(-24 - 4\lambda) = 0$$

$24 - 8\mu = 0$; $\quad -24 = 4\,\lambda$;

$\mu = 3 \quad \lambda = -6$

(d) to be coplanar $(\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c} = 0$

Let $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ $\quad \boldsymbol{b} = \boldsymbol{i} + \boldsymbol{j} + d\boldsymbol{k}$ $\quad \boldsymbol{c} = 3\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}$

$$(\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c} = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 1 & d \\ 3 & -4 & 1 \end{vmatrix} = 0$$

$2(1 + 4d) + 1(1 - 3d) + (-4 - 3) = 0$

$-4 + 5d = 0, \quad d = {}^{4}/_{5}$

(e) $\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 4 & 5 & -1 \\ 1 & -4 & 5 \end{vmatrix} = \boldsymbol{i}(25-4) - \boldsymbol{j}(20+1) + \boldsymbol{k}(-16-5) = 21(\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k})$

Since $\boldsymbol{d}$ is perpendicular to $\boldsymbol{a}$ and $\boldsymbol{b}, \boldsymbol{d}$ is parellel to $\boldsymbol{a} \times \boldsymbol{b}$

$\therefore \boldsymbol{d} = \lambda \times 21(\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k})$

But $\boldsymbol{d} \cdot \boldsymbol{c} = 21 = 21\lambda(\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}) \cdot (3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k})$

$\Longrightarrow 21\lambda \times 3 - 21\lambda + 21\lambda = 21, \quad \lambda = {}^{1}\!/_{3}$

$\boldsymbol{d} = 7(\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k})$

(4). (a) $(AB)^{-1} = B^{-1}A^{-1} = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$

$$B(B^{-1}A^{-1}) = B\begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$$

$$(BB^{-1})A^{-1} = \begin{pmatrix} 2 & 4 \\ 7 & 8 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$$

$$IA^{-1} = \begin{pmatrix} 2 & 4 \\ 7 & 8 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 2 & 4 \\ 7 & 8 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 4-4 & 6-8 \\ 14-8 & 21-6 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 6 & 5 \end{pmatrix}$$

(b) Symmetric matrix : $B = \frac{1}{2}\,[A + A^T] = \frac{1}{2}\left\{\begin{bmatrix} 4 & 5 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 5 & 3 \end{bmatrix}\right\} = \begin{bmatrix} 4 & 3 \\ 3 & 3 \end{bmatrix}$

Skew symmetric matrix : $C = \frac{1}{2}\,[A - A^T] = \frac{1}{2}\left\{\begin{bmatrix} 4 & 5 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ 5 & 3 \end{bmatrix}\right\} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

(c) A is hermition $a_{ij} = \overline{a_{ji}}$

$$A = \begin{bmatrix} 3 & x+2i & yi \\ 3-2i & 0 & 1+2i \\ yi & 1-xi & -1 \end{bmatrix}$$

$a_{12} = \overline{a_{21}}$	$a_{13} = \overline{a_{31}}$	$a_{23} = \overline{a_{32}}$
$x + 2i = \overline{3-2i} = 3 + 2i$	$yi = -yi$	$1 + zi = \overline{1 - xi} = 1 + x$
$x = 3$	$y = 0$	$1 + zi = 1 + 3i$
		$z = 3$

(d) $AB = \begin{bmatrix} 2 & 0 & 5 \\ 1 & -2 & 4 \\ -3 & 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 8 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 29 & -5 \\ 6 & -6 \\ 32 & -5 \end{bmatrix}$

exists, as No. of columns in A is equal to no of rows in B

$BA = \begin{bmatrix} 2 & 0 \\ 8 & 1 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 2 & 0 & 5 \\ 1 & -2 & 4 \\ -3 & 1 & -1 \end{bmatrix}$

BA Does not exist, as No. of columns in B is not equal to no of columns in A

(e) $2x - 5y + 2z = 2$

$x + 2y - 4z = 5$

$3x - 4y - 6z = 1$

$$\underbrace{\begin{pmatrix} 2 & -5 & 2 \\ 1 & 2 & -4 \\ 3 & -4 & -6 \end{pmatrix}}_{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$$

$|A| = 2(-12 - 16) + 5(-6 + 12) + 2(-4 - 6)$

$= -2 \times 28 + 5 \times 6 - 2 \times 10 = -46 \neq 0$

$$x = \frac{|A^{(1)}|}{|A|} = \begin{vmatrix} 2 & -5 & 2 \\ 5 & 2 & -4 \\ 1 & -4 & -6 \end{vmatrix} \times \frac{-1}{46} = (-230) \times \frac{-1}{46} = 5$$

$$y = \frac{|A^{(2)}|}{|A|} = \begin{vmatrix} 2 & 2 & 2 \\ 1 & 5 & -4 \\ 3 & 1 & -6 \end{vmatrix} \times \frac{-1}{46} = (-92) \times \frac{-1}{46} = 2$$

$$z = \frac{|A^{(3)}|}{|A|} = \begin{vmatrix} 2 & -5 & 2 \\ 1 & 2 & 5 \\ 3 & -4 & 1 \end{vmatrix} \times \frac{-1}{46} = (-46) \times \frac{-1}{46} = 1$$

(5). (a) $1 \leq i \leq 4 \qquad 1 \leq j \leq 4 \qquad$ take $i = 1$

$$|A| = -x\begin{vmatrix} -x & 1 & 0 \\ 0 & -x & 1 \\ -c_1 & -c_2 & -c_3 - x \end{vmatrix}(-1)^{1+1} + 1\begin{vmatrix} 0 & 1 & 0 \\ 0 & -x & 1 \\ -c_0 & -c_2 & -c_3 - x \end{vmatrix}$$

$$+0\begin{vmatrix} 0 & -x & 0 \\ 0 & 0 & 1 \\ -c_0 & -c_1 & -c_3 - x \end{vmatrix}(-1)^{1+3} + 0\begin{vmatrix} 0 & -x & 1 \\ 0 & 0 & -x \\ -c_0 & -c_1 & -c_2 \end{vmatrix}(-1)^{1+4}$$

$$|A| = -x\left\{-x\begin{vmatrix} -x & 1 \\ -c_2 & -c_3 - x \end{vmatrix}(-1)^{1+1} + 1\begin{vmatrix} 0 & 1 \\ -c_1 & -c_3 - x \end{vmatrix}(-1)^{1+2} + 0\begin{vmatrix} 0 & -x \\ -c_1 & -c_2 \end{vmatrix}(-1)^{1+3}\right\}$$

$$-\left\{0\begin{vmatrix} -x & 1 \\ -c_2 & -c_3 - x \end{vmatrix}(-1)^{1+1} + 1\begin{vmatrix} 0 & 1 \\ -c_0 & -c_3 - x \end{vmatrix}(-1)^{1+2} + 0\begin{vmatrix} 0 & -x \\ -c_0 & -c_2 \end{vmatrix}(-1)^{1+3}\right\}$$

$|A| = -x[-x(x(c_3 + x) + c_2) - (0 + c_1)] - c_0$

$= x^2(xc_3 + x^2 + c_2) + xc_1 + c_0$

$|A| = x^4 + x^3c_3 + x^2c_2 + xc_1 + c_0$

(b) (i) $|A| = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1+a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1+c \end{vmatrix} \xrightarrow[\substack{C_3 \to C_3 - C_1 \\ C_4 \to C_4 - C_1}]{C_2 \to C_2 - C_1} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & a & 0 & 0 \\ 1 & 0 & b & 0 \\ 1 & 0 & 0 & c \end{vmatrix}$

$\xrightarrow[\substack{R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1}]{R_2 \to R_2 - R_1} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \end{vmatrix} \to a\,b\,c \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \to |A| = a\,b\,c$

(ii) $|B| = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} \xrightarrow[C_2 \to C_2 - C_3]{C_1 \to C_1 - C_2} \begin{vmatrix} a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \\ b-a & c-b & a+b \end{vmatrix}$

$\to (a-b)\,(b-c) \begin{vmatrix} 1 & 1 & c \\ a+b & b+c & c^2 \\ -1 & -1 & a+b \end{vmatrix}$

$\xrightarrow{C_2 \to C_2 - C_1} (a-b)\,(b-c) \begin{vmatrix} 1 & 0 & c \\ a+b & c-a & c^2 \\ -1 & 0 & a+b \end{vmatrix}$

$\to (a-b)\,(b-c)(c-a) \begin{vmatrix} 1 & 0 & c \\ a+b & 1 & c^2 \\ -1 & 0 & a+b \end{vmatrix}$

$\xrightarrow{R_1 \to R_1 + R_3} (a-b)\,(b-c)(c-a) \begin{vmatrix} 0 & 0 & a+b+c \\ a+b & 1 & c^2 \\ -1 & 0 & a+b \end{vmatrix}$

$\to (a-b)\,(b-c)(c-a)(a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ a+b & 1 & c^2 \\ -1 & 0 & a+b \end{vmatrix}$

$|B| = (a-b)\,(b-c)(c-a)(a+b+c)$

(c) (i) $\begin{vmatrix} 0 & a & b & c \\ a & 0 & c & b \\ b & c & 0 & a \\ c & b & a & 0 \end{vmatrix} = \frac{1}{bc\ acab} \begin{vmatrix} 0 & bca & acb & abc \\ a & 0 & ac^2 & ab^2 \\ b & bc^2 & 0 & a^2b \\ c & b^2c & a^2c & 0 \end{vmatrix} = \frac{abcabc}{bc\ acab} \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & c^2 & b^2 \\ 1 & c^2 & 0 & a^2 \\ 1 & b^2 & a^2 & 0 \end{vmatrix}$

$= \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & c^2 & b^2 \\ 1 & c^2 & 0 & a^2 \\ 1 & b^2 & a^2 & 0 \end{vmatrix}$

(ii) $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = \begin{vmatrix} a & b & c \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} + \begin{vmatrix} b & c & a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$

$= \begin{vmatrix} a & b & c \\ b & c & a \\ c+a & a+b & b+c \end{vmatrix} + \begin{vmatrix} a & b & c \\ c & a & b \\ c+a & a+b & b+c \end{vmatrix}$

$+ \begin{vmatrix} b & c & a \\ b & c & a \\ c+a & a+b & b+c \end{vmatrix} (\to 0) + \begin{vmatrix} b & c & a \\ c & a & b \\ c+a & a+b & b+c \end{vmatrix}$

$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} + \begin{vmatrix} a & b & c \\ b & c & a \\ a & b & c \end{vmatrix} (\to 0) + \begin{vmatrix} a & b & c \\ c & a & b \\ c & a & b \end{vmatrix} (\to 0)$

$+ \begin{vmatrix} a & b & c \\ c & a & b \\ a & b & c \end{vmatrix} (\to 0) + \begin{vmatrix} b & c & a \\ c & a & b \\ c+a & a+b & b+c \end{vmatrix}$

$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} + \begin{vmatrix} b & c & a \\ c & a & b \\ c & a & b \end{vmatrix} (\to 0) + \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix} (\to R_1 \leftrightarrow R_3)$

$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} - \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} (\to R_2 \leftrightarrow R_3)$

$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} - \left(- \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \right) = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

(iii) $\begin{vmatrix} bcd & cda & dab & abc \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{vmatrix} = \frac{1}{abcd} \begin{vmatrix} abcd & bcda & cdab & dabc \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \\ a^4 & b^4 & c^4 & d^4 \end{vmatrix}$

$= \frac{abcd}{abcd} \begin{vmatrix} 1 & 1 & 1 & 1 \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \\ a^4 & b^4 & c^4 & d^4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \\ a^4 & b^4 & c^4 & d^4 \end{vmatrix}$

(d) (i) $\begin{vmatrix} t-4 & 3 \\ 2 & t-9 \end{vmatrix} = 0$

$(t-4)(t-9) - 6 = 0$

$t^2 - 113t + 30 = 0$

$(t-10)(t-3) = 0$

$t = 10$ or $t = 3$

(ii) $\begin{vmatrix} t-1 & 4 \\ 3 & t-2 \end{vmatrix} = 0$

$\begin{vmatrix} t-1 & 4 \\ 3 & t-2 \end{vmatrix} = 0$

$(t-1)(t-2) - 12 = 0$

$t^2 - 3t - 10 = 0$

$(t+2)(t-5) = 0$